DAR model Overleaf

**Levels or First Difference? Log-transforming data?**

It is argues by (XXX) that due to

**The DARMA-X Model**

**Extending the DAR Model**

From the stylised facts, we have seen that modelling conditional mean and volatility together is an important step towards capturing the dynamics of electricity prices. One specification is the Double AR() – also called the DAR() – model, it combines an autoregressive process with conditional heteroskedasticity, and has been studied extensively in, amongst other, (XXX), (XXX) and (XXX). The DAR() model is defined as,

, where , , and . From derivations in Appendix XXX and Jiang et al. (2019), the strict stationarity and moment conditions for the DAR() with Gaussian innovations, , are tabualted in Table XXX

|  |  |  |
| --- | --- | --- |
| Model: DAR(1) with | | |
| **Condition** | **Constraint** | **Region in Fig XXX** |
| Strict Stationarity |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

Since implies it is a sufficient, but not necessary, condition to perform 1-step forecasts.

As we see in Fig XXX the autoregressive coefficeont is allowed to be , which contrasts starkly with the classicial AR() model that requires for weak stationarity. ARIMA-type models deal with this problem by performing estimation on the differenced series. However, this approch removes valuable information about the stochastic process, and is therefore suboptimal. As argued in (Ling 2004), the reason why the DAR() is stationary in the precense of unit-roots (or even explosive roots), is that – for – the conditional variance can control the log-likelihood function, score function and information matrix in such a way that they are bounded.

Ling 2014 shows that this holds for many different, and heavy-tailed, distributions of .

The DAR() is

Compared ARIMA

**Drift Criteria**

A math equations and formulas

Description automatically generated with medium confidence

**DAR Moments**

**First Order Moment**

Let be the drift function that bounds :

Some usefull rules:

* and are independent for all
* Multiplicativeness:
* Subadditivity 1:
* Subadditivity 2:

Since we have restricted and , then,

Using Subadditivity 2 and Multiplicativeness we get that . Therefore,

For , then and for , then ?

With this can be written as,

, where the constnat is defined as . To mimic a first order autoregressive process, this can be reformulated as,

, with such that by definition. Therefore, it is clear that if the drift criterion holds and the process is bounded such that .

**DARMA Moments**

**Second Order Moment**

If , the Euclidian norm corresponds to the drift criterion that bounds the aboslute value on vector form,

Then the drift criterion that bounds the second order moments is,

With we consider the drift function ,

Examining first first term:

All the cross-terms cancel out since ,

We have that therefore and are independent,

We will now derive ,

Using the distributive property,

Then the taking the expectation yields,

Since and for from the assumption of independence. If then sum converges to a infinite geometric series,

Inserting back into term 1 yields,

Examining second term:

All the cross-terms cancel out since ,

Using the arguments as before reveals that the solution to is independent of the lag since and , yielding the same solution for any . Therefore,

Inserting back into term 2 yields,

Putting the two terms together we get,

Defining constants and as and ,

Using that this can be rewritten as,

, where . Further, to minic an AR(1) process we have,

With and by defintion. Assumming , we can reduce to the following,

Therefore, if the drift criterion holds and the second order moment is bounded.

**First Order Moment**

With we consider the drift function ,

Some usefull rules:

* and are independent for all
* Multiplicativeness:
* Subadditivity 1:
* Subadditivity 2:

Using Subadditivity 2,

Examining the first term:

With , since is independent of then . Therefore, also ,

Examining the first term:

**DARMA-X Moments**

For we have that and for we have that

Assuming .

**Second Order Moment**

With we consider the drift function

Examining first term:

All the cross-terms cancel out since ,

Examining second term:

Same answer, except variables are lagged by one more,

Putting the two terms together we get,

Defining constants , and as , and ,

Using that can be rewritten as,

, where . We can further rewrite the expression to mimic an AR(1) process,

With and by construction. Assumming , we can reduce to the following,

Therefore, if , and is stationary, the drift criterion holds and the second order moment is bounded.

**Quasi Maximum Likelihood Estimation**

The Gaussian maksimum likelihood function is given as,

Conditions for consistent estimation.

Fff